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A study on parabolic mass distribution

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Abstract

Civil Engineering structures have to withstand natural environmental forces like wind, earthquake forces and wave forces, along with loads that they are designed to resist. All this environmental forces are random and dynamic in nature. Therefore the response of the structure is also dynamic and that is what causes the unsafe and uncomfortable conditions. Therefore there is always a need for some sort of control of response of structure. This project aims at studying both methods of the Tuned Mass Dampers. It has been well established that Single tuned mass damper (STMD) and Multiple tuned mass damper (MTMD) are effective in reducing the response of the structure. The project aims and study of two devices, the Single Tuned Mass Damper and Multiple Tuned Mass Damper using new control strategy. The tuned mass dampers, consisting of one larger mass block (*i.e.* one larger tuned mass damper) and one smaller mass block (*i.e.* one smaller tuned mass damper), referred in this report as the STMD, have been studied to seek for the mass dampers with high effectiveness and robustness for the reduction of the undesirable vibrations of structures under the ground acceleration. Multiple tuned mass dampers (MTMD) consisting of many active tuned mass dampers (TMDs) with uniform distribution of natural frequencies have been, proposed to attenuate undesirable oscillations of structures under the ground acceleration.

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Introduction

Civil Engineering structures have to withstand environmental forces like wind, earthquake forces and wave forces along with loads that they are designed to resist. All this environmental forces are random and dynamic in nature. Therefore the response of the structure is also dynamic and that is what causes the unsafe and uncomfortable conditions. Therefore there is always a need for some sort of control of response of structure. The fact is more important in present times due to following factors:

1. Increased flexibility: it is now a necessity and trend to use tall, long or in general more flexible structures. There is also a growing tendency to use lighter and more flexible construction materials. These factors promote the idea of control of vibrations of structure.

2. Increased safety levels: As structure becomes more complex, costly and as it serves more critical function, it demands higher safety levels.

3. Stringent performance requirements: Structures are required to respond to the forces acting on them within the safety limits. Hence for environmental loads, which are random and dynamic in nature, more stringent safety limits are generally set, which demand for control of vibrations of the structure. Due to the above listed reasons, the concept of structural perception using control systems is not only becoming increasingly popular but it is becoming almost a necessity in modern days. The Tuned Mass Damper is a classical engineering device that is used for vibration control. It consists of mass, a spring and a damper, which is attached to the main structure Fig 1.

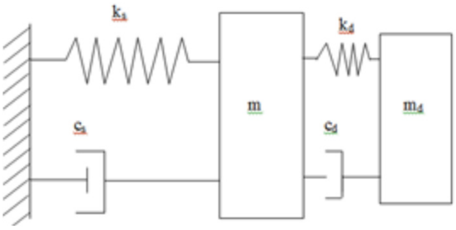

Fig 1 The Single tuned mass damper

The mechanism of suppressing structural response by attaching tuned mass damper to the structure is to transfer the vibration energy of the structure to the tuned mass damper and to dissipate the energy at the damper of TMD. In order to enlarge the dissipation of energy in TMD, it is essential to tune the natural frequency of TMD to that of structural motion. The TMD has many advantages like compactness, reliability, efficiency and low maintenance cost as compared to other damping devices. Hence, it is widely used in civil engineering structures. Single tuned mass dampers (STMD) have proved to be very sensitive even to the small offset in tuning ratio when it is optimally designed. This is the greatest disadvantage of STMD. This is due to following reasons. Errors in predicting or identifying the natural frequency of the structure and also the error in fabricating a TMD are inevitable to some degree. Some structures have nonlinear properties even in small amplitude range due to contribution of secondary members. Therefore, in practical design the optimum values of parameters of TMD are not chosen. The damping of the TMD is intentionally made higher than the optimal value such that TMD become less sensitive to tuning errors. This results increase the mass of TMD to meet the design requirement. All these uncertainties can be reduced by use of Multiple Tuned Mass Dampers (MTMD). Use of MTMD has been proposed to increase the robustness of the vibration control system to various uncertainties in the structures and/or TMD. The basic configuration of MTMD is the large number of small oscillators whose natural frequencies are distributed around the natural frequency of the controlled mode of structure. It is now well established that an optimal MTMD is more effective and robust than optimal STMD.

Literature review

Tuned mass dampers (TMD) are widely used to control the vibrations in civil engineering structures. Although TMDs are effective in reducing the vibrations caused by stationary excitation forces, their performance to suppress seismic response is limited. This inefficiency is due to the fact that TMDs usually need some time interval before it becomes fully effective because they are initially at rest, while the strongest seismic ground motion is often observed at the earlier stage of an earthquake. Another drawback is that TMDs are sensitive to tuning error. Employing more than one tuned mass damper with different dynamic characteristics has then been proposed to further improve the effectiveness and robustness of the TMD. The multiple tuned mass dampers (MTMD) with the distributed natural frequencies were proposed by Xu and Igusa (1991) and also studied by, Abe and Fujino (1994), Abe and Igusa (1995), Bakre and Jangid (2004),Chen and Wu (2001), Gu *et al*. (2001), Han and Li (2005), Jangid (1995), Joshi and Jangid (1997), Kareem and Kline (1995), Kamiya *et al*.(1992), Li (2000), Li and Liu (2004), Li and Qu (2000), Lin (2005), Park (2001), Wang (2005), Yau (2004), Yamaguch and Hampornchai (1993). The MTMD is shown to possess better effectiveness and higher robustness in mitigating the oscillations of structures with respect to a single TMD.

Studies of TMD

Likewise, the dual-layer multiple tuned mass dampers, referred to as the DL-MTMD consisting of one larger tuned mass damper and several smaller tuned mass dampers with the total number of tuned mass damper units being the arbitrary integer and with the uniform distribution of natural frequencies have been further proposed by Li (2005) to seek for the mass dampers with high effectiveness and robustness for the reduction of the undesirable vibrations of structures under the ground acceleration. The numerical results indicate that the DL-MTMD can render better effectiveness and higher robustness to the change in the natural frequency tuning (NFT), in comparison with the multiple tuned mass dampers (MTMD) with equal total mass. In fact the DL-MTMD will degenerate into the double tuned mass damper when the total number of the smaller tuned mass damper units in the DL-MTMD is set to be equal to unity. The investigations by Li (2005) have manifested that the DL-MTMD has a little better effectiveness with respect to the DTMD, but they practically reach the same level of robustness to the change in the natural frequency tuning (NFT). The DTMD consists of one larger mass block (larger tuned mass damper) and one smaller mass block (i.e. smaller tuned mass damper), thus implying that it is significantly simpler to manufacture the DTMD in comparison with the DL-MTMD. With a view to the engineering design and practical applications, it is imperative and of practical interest to carry on further investigations on the DTMD.

Active TMDs can be effective in reducing seismic response because the TMD amplitude can be increased much faster through the use of the actuators. They can also be more robust to tuning errors with the appropriate use of feedback. Therefore, active TMDs have attracted broad research interest and various control algorithms have been developed Yang *et al*. (1987), Spencer *et al*. (1994), Chang and Yang (1994). Because of their efficiency and compactness, active TMDs have been successfully designed and installed in full scale Kobori (1991).

Yao (1972) made an attempt to stimulate interest among structural engineers in the application of control theory in the design of civil engineering structures. This has been concluded that much more work is needed in order to apply the concept of structural control to complicated structures such as extremely tall buildings or long bridges subjected to uncertain dynamic loads such as wind and earthquake excitations.

Modern control theories that were developed during the past decade have been successfully applied to the control of the trajectory and motions of space vehicles as well as aeronautical systems. Recently, the control theory has also been applied to reduce the vibration of civil engineering structures Yang (1975). The major difficulty to be encountered is that most civil engineering structures have been very heavy.

Experiments on active control of Seismic Structures have been presented by Chung *et al* (1998) in which the first phase of a comprehensive experimental study concerning the possible application of active control to structures under seismic excitations is discussed. The experiment consisted of a single degree of freedom model structure, controlled using prestressing tendons connected to the servo hydraulic actuators. An optimal closed loop control scheme using a quadratic performance index was employed to reduce the response of structure under base motion generated by a large scale seismic simulator. Using a carefully designed, fabricated, and calibrated experimental setup the correlation between the analytical and experimental results was studied. Based on similitude relations, the experimental results obtained for the model structure was extrapolated to the full scale structures are analyzed.

Reinhom *et al*. (1987) presented a methodology for the shape control of structures undergoing inelastic deformations through the use of an active pulse force system. To avoid the large deformation in structures like tall buildings, long bridges and offshore platforms external forces are applied to the structure through cables, air jets, or other devices in order to ensure that the deformations are kept below the limits set for serviceability at all times.

Yang (1975) investigated the feasibility of optimum active control theory for controlling the motion and vibration of civil engineering structures. It is assumed that the structural system can be discredited, such that the equation of motion can be described by a system of ordinary differential equations. The effectiveness of the control system is measured by a performance index. The optimal control law, which minimizes the performance index, is a linear feedback control. The optimal control forces are obtained by solving a matrix Ricatti equation. Moreover, the feasibility of implementing the active control by means of active dampers and servomechanism is considered there.

Abe (1996) is also proposing a rule based on control algorithm for active TMDs. First, perturbation solutions of the linear quadratic regulator (LQR) feedback gains for the active TMD system are derived. Using these solutions interaction of the TMD and the actuator force is discussed in detail. The algorithm consisted of two parts: (1) a variable gain displacement feedback control (2) a variable TMD damping control. The first one is applied when the TMD amplitude is small to make the TMD more effective, and the second one is applied when the TMD amplitude is large to dissipate the energy.

Sarbjeet *et al* (1998) presented a control strategy based on the combination of feed forward and feedback gain controls (an open-closed loop) for the reduction of the displacement response of the shear frame model of tall buildings to random ground motion which is represented by double filtered white noise.

Methodology

Symmetrical mass distribution schemes

Studies conducted on Tuned mass dampers have focused mainly on systems having a constant mass ratio for each TMD. Yamaguchi and Harnpornchai[3] have investigated the performance of MTMD's for a constant mass distribution. The mass distribution is, however a very important parameter and the structural response can be controlled by using a proper mass distribution scheme. The main idea is to transform two peak responses to a one peak response and flatten out the peak by controlling the mass distribution along with other parameters like damping ratios, frequency range, and the number of tuned mass dampers.

The effect of mass distribution on structural response of a structure MTMD system is studied. Various types of frequency distributions with zero offset frequency are used. The damping ratio is kept constant and structural damping is taken as 1 percent. The total mass ratio is kept constant at 1% of structural mass. The mass distribution examined is parabolic and linear.

Parabolic Mass Distribution

A parabolic mass variation of the form

$$\mu_i = \left\{ \left[1 - \left(\frac{n+1}{2} \right)^2 \right] a - \left(i - \frac{n+1}{2} \right)^2 \right\} \quad 3.1$$

where n is the total number of TMD's and 'a' is a constant whose value is taken to be 30 for the analysis. The idea here is to have maximum mass at the centre so as to damp out central peaks. The entire distribution can be adjusted so as to damp out all the secondary peaks as well.

The structure considered is mass excited one after Yamaguchi and Harnpornchai[3]. The results are almost comparable to those obtained by them. The parameters that have been varied are the damping ratio of individual TMD's, the frequency range of the TMD's, and the total number of TMD's. One is interested in a symmetrical response curve with a flat peak.

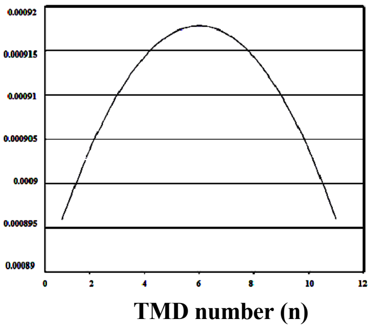

Fig. 2 Mass distribution for a= 30

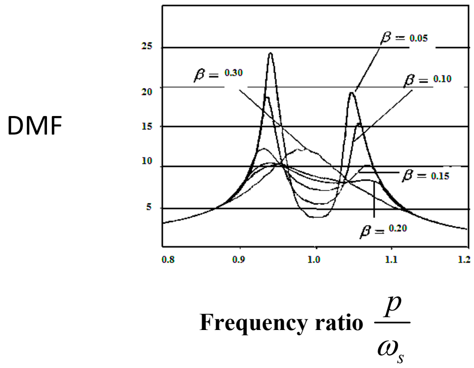

Fig. 3 Effect of varying the frequency spacing parameter β

It can be seen from the above figure that the controlled structural response is transformed from a two peak response to a one peak characteristic with increasing frequency range of MTMD. There is therefore a value of the frequency range for which the response curve is flat and peak is minimum and the value at which this occurs is β □= 0.20. The maximum DMF is about 10. The flatness of the response curve and width of the frequency region have not however been affected significantly as compared to Yamaguchi and Harnpornchai[3], where the corresponding values are about 11 and 0.17 respectively.

Optimum Parameters for parabolic mass distribution

Three types of frequency distribution are considered for the analysis as shown in the Figure. Type I is linear whereas Type II and Type III are varying.

Frequency distribution (Type I)

The parameters were varied systematically and preliminary values for each parameter were arrived by choosing the one that gave the least value of maximum DMF and a flat response curve. A mass ratio of one percent has been assumed throughout the analysis. Graphs below compare the optimum DMF curves for three different cases, viz, parabolic mass, constant stiffness and constant mass for three types of frequency distributions as shown in Fig.5.3. Structural damping is taken to be 1%.

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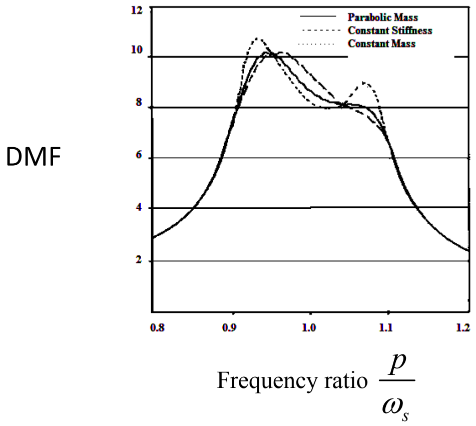

Fig.4 Optimum parameter curve for linear frequency distribution.

Optimum parameters for linear mass distribution

A linear mass distribution of the form

$$\mu_k = \mu_{\frac{(n+1)}{2}} - \left| d\mu\left(\frac{n+1}{2} - k\right)\right|$$

(k=1, 2, 3……n) (3.19)

where $d\mu = \frac{4\mu_r}{(n+1)^2}$

has been used. The idea here is to have maximum mass at the centre, so as to damp out central peaks and the mass of all other TMD's decreases linearly towards both the ends. Optimum parameters are than obtained for three types of frequency distributions. A total mass ratio of 1% has been assumed; also structural damping is taken to be 1%.

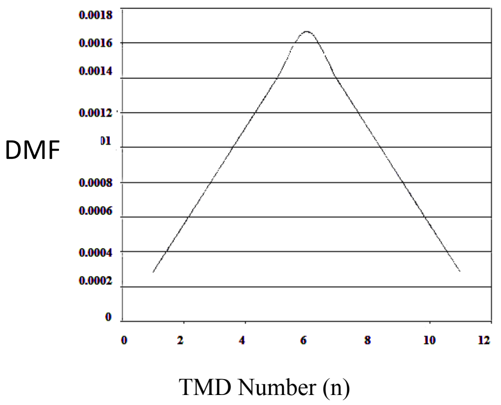

Fig.6 Linear mass distribution

Frequency distribution (Type I)

The parameters were varied systematically and preliminary values for each parameter were arrived by choosing the one that gave the least value of maximum DMF and a flat response. Again the mass ratio of one percent has been assumed. Graphs below compare the optimum DMF curves for three different cases, viz, linear mass, constant stiffness and constant mass for three types of frequency distributions as shown in Fig.7.

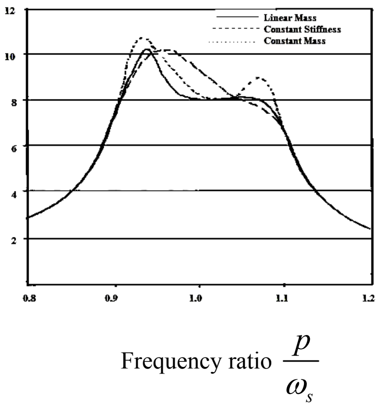

Fig. 7 Optimum parameter curve for linear frequency distribution

Frequency distribution (Type II and Type III)

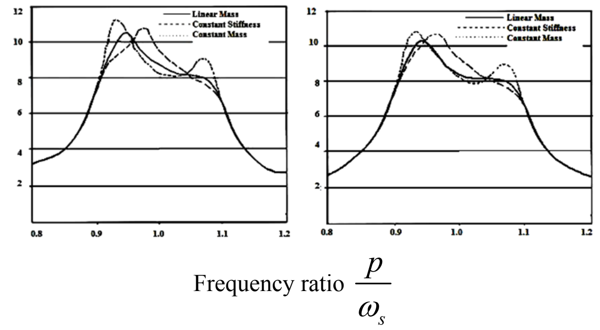

Fig. 5 Optimum parameter curve for frequency distribution of Type II and Type III

	Damping ratio ξ_d			Tuning frequency ratio γ			Spacing parameter β			DMF		
	Type I	Type II	Type III	Type I	Type II	Type III	Type I	Type II	Type III	Type I	Type II	Type III
Parabolic mass	0.0155	0.0172	0.1656	0.99875	0.99971	0.9982	0.2013	0.1986	02003	10.212	10.458	10.354
Constant Stiffness	0.0196	0.0163	0.0185	0.99642	1.0141	1.0023	0.1651	0.1751	0.1862	10.425	10.876	10.521
Constant Mass	0.0201	0.0156	0.0166	0.99321	1.0122	1.0081	0.1723	0.1353	0.1985	10.912	11.223	11.028

Table 1: Optimum values for three types of frequency distributions (Parabolic Mass)

Optimum parameters for linear mass distribution

A linear mass distribution of the form

$$\mu_k = \mu_{\frac{(n+1)}{2}} - \left| d\mu\left(\frac{n+1}{2} - k\right)\right|$$

(k=1, 2, 3……n) (3.19)

where $d\mu = \frac{4\mu_r}{(n+1)^2}$

has been used. The idea here is to have maximum mass at the centre, so as to damp out central peaks and the mass of all other TMD's decreases linearly towards both the ends. Optimum parameters are than obtained for three types of frequency distributions. A total mass ratio of 1% has been assumed; also structural damping is taken to be 1%.

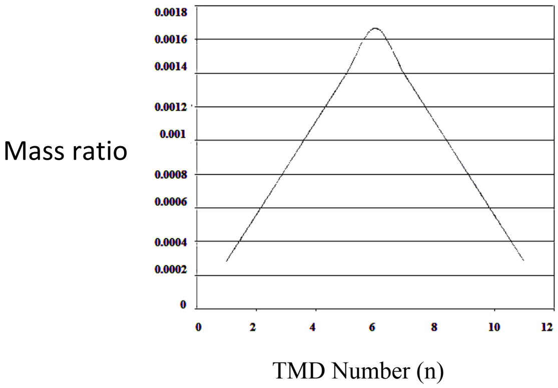

Fig.6 Linear mass distribution

Frequency distribution (Type I)

The parameters were varied systematically and preliminary values for each parameter were arrived by choosing the one that gave the least value of maximum DMF and a flat response. Again the mass ratio of one percent has been assumed. Graphs below compare the optimum DMF curves for three different cases, viz, linear mass, constant stiffness and constant mass for three types of frequency distributions as shown in Fig.7.

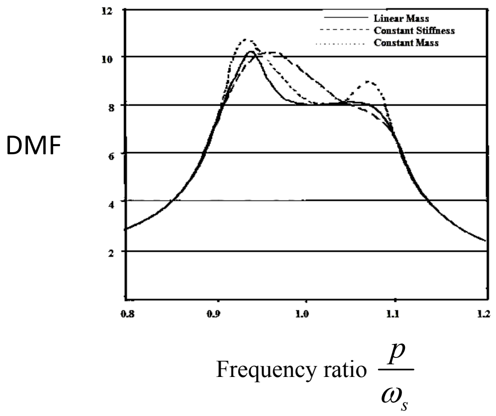

Fig. 7 Optimum parameter curve for linear frequency distribution

Frequency distribution (Type II and Type III)

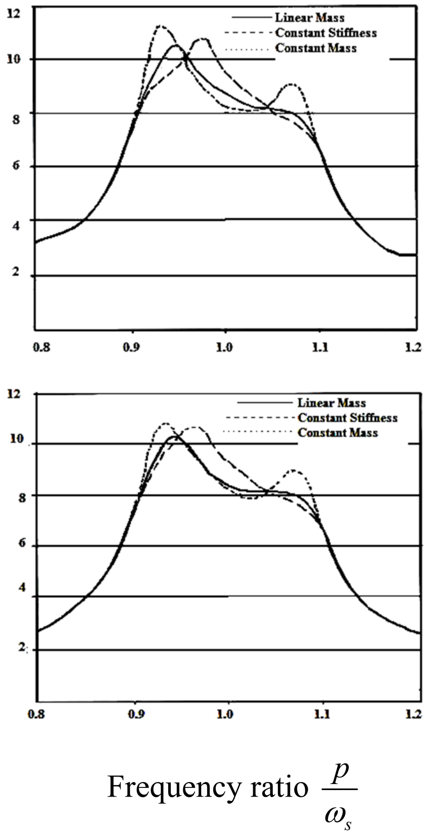

Fig. 8 Optimum parameter curve for frequency distribution of Type II and Type III

Result and Conclusions

The whole purpose to draw the above graphs was to compare the results for various types of frequency distributions. It can be easily concluded that linear frequency distribution or Type I is most effective followed by Type III and then Type II. Thus an optimally designed

MTMD system with uniform distribution frequencies is more effective than other non-uniform distributions. Also it can be seen that it's not possible to obtain a perfectly flat response curve even after using symmetrical mass distributions. There is not much improvement in the value of max DMF for the three cases considered.

	Damping ratio ξ_d			Tuning frequency ratio γ			Spacing parameter β			DMF		
	Type I	Type II	Type III	Type I		Type I	Type II	Type III	Type I		Type I	Type II
Linear Mass	0.0152	0.0159	0.0168	0.99882	Linear Mass	0.0152	0.0159	0.0168	0.99882	Linear Mass	0.0152	0.0159
Constant Stiffness	0.0196	0.0163	0.0185	0.99642	Constant Stiffness	0.0196	0.0163	0.0185	0.99642	Constant Stiffness	0.0196	0.0163
Constant Mass	0.0201	0.0156	0.0166	0.99321	Constant Mass	0.0201	0.0156	0.0166	0.99321	Constant Mass	0.0201	0.0156

Table 1: Optimum values for three types of frequency distributions (Linear Mass)

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